Radial Void Fraction Correlation for Annular Packed Beds

Gary E. Mueller
Nuclear Engineering, University of Missouri-Rolla, Rolla, MO 65409

Annular packed beds (APBs) involving the flow of fluids are used in many technical and engineering fields. Investigations using APBs have involved chemical reactors (Yagi and Kunii, 1960; Cheng and Hsu, 1986; Song et al., 1994), heat exchangers and thermal systems (Rimkevicius et al., 1991; Choi and Kulacki, 1993; Kamiuto et al., 1993; Heggs et al., 1994, 1995; Sodre and Parise, 1998), fusion reactor blankets (Dean, 1983; McWhirter et al., 1998), and nuclear particle-bed reactors (Powell and Horn, 1986; Horn et al., 1986, 1987; Ludewig et al., 1996). It is well known that the wall in a packed bed affects the void fraction distribution. Correlations that predict the radial void fraction for packed beds in cylindrical containers have been formulated from experimental and analytical data (Ridgway and Tarbuck, 1968; Martin, 1978; Cohen and Metzner, 1981; Govindarao and Froment, 1986; Kubie, 1988; Govindarao et al., 1990; Mueller, 1991, 1992).

For accurate transport modeling of the hydrodynamics, heat and mass transfer in packed beds, it is important to include the variation of the void fraction (Vortmeyer and Schuster, 1983; Cheng and Hsu, 1986; Vortmeyer and Haidegger, 1991; Borkink and Westerterp, 1994; Papageorgiou and Froment, 1995; Bey and Eigenberger, 1997; Subagyo and Brooks, 1998). Since APBs have two walls that can simultaneously affect the void fraction distribution, it is essential to include this void fraction variation in transport models (Cheng and Hsu, 1986; Sodre and Parise, 1998), who used two exponential equations that are piecewise continuous in an APB to model the radial void fraction. These exponential equations do not model the well-known damped oscillations of the radial void fraction. Therefore, the APB near wall velocity profiles by Sodre and Parise (1998) do not exhibit damped oscillations such as those obtained by Subagyo and Brooks (1998), where an accurate radial void fraction model was used in the analysis. To more precisely model the hydrodynamic, heat and mass transfer in APBs, a correlation that closely predicts the void fraction distribution is required. Presently, there is no such published correlation.

The purpose of this study is to calculate radial void fraction distributions of equal sized spheres in annular packed beds and to obtain a radial void fraction correlation from these distributions that can be incorporated in APB transport models.

Void Fraction Development

Since there are no published data on radial void fraction distributions in APBs, the objective of the void fraction development is to use analytical/numerical methods to obtain these distributions. This is accomplished by expressing the local radial void fraction in any radial annular segment in terms of the solid volume contributions coming from spheres with center coordinates at positions within a particle radius on either side of the annular segment. Mueller (1992) gives the procedure used to determine the local void fraction for the radial annular segments in cylindrical containers. This procedure for determining the local void fraction is also easily applied to APBs. In the case of APBs there is only one type of intersecting geometry which can occur for a sphere and the radial annular segment that intersects the sphere (Mueller, 1992). That geometry is for spheres with centers that are at radial locations greater than a particle radius from the center of the APB. The other three geometries described by Mueller (1992) do not apply for this investigation since they are for spheres that are at locations less than a particle radius from the center of the container and this region does not exist in the APBs.

The local radial void fraction is then calculated from the total sum of solids from the spheres in the radial annular segments. This process is performed for all the annular segments at a particular radial position to obtain the local radial void fraction. The result is an accurate void fraction distribution as a function of the radial position in the packed bed (Mueller, 1992).

The procedure by Mueller (1992) for calculating the radial void fraction distributions requires knowledge of the sphere center coordinates. The center coordinates for this study of the equal-sized spheres in an APB are determined from an analytical/numerical method given by Mueller (1997). The method numerically constructs packed beds of identical spheres in right circular cylindrical containers by using a sequential packing model. The sequential model calculates the center coordinates of each newly added sphere. The method can easily be applied to APBs since they consist of concentric right circular cylindrical containers. For this APB study, the Bennett model (Mueller, 1997) is used as the sequential packing procedure. This procedure always selects the lowest vertical stable position to place a sphere. Spheres are placed on three spheres (inner spheres) or on two spheres and a wall (wall spheres), as long as the position is a stable location (Mueller, 1997). The center coordinates obtained from the sequential numerical procedure are used in the void fraction development to calculate the radial void fraction distributions.

Void Fraction Correlation

The radial void fraction correlation is formulated from the radial void fraction distributions that are obtained from the void fraction development. The correlation is restricted to randomly packed beds in annular cylindrical containers of outside diameter (m) D_o, inside diameter (m) D_i, equivalent diameter (m) $D_e = D_o - D_i$, consisting of equal-sized spheres of diameter (m) d, with diameter aspect ratios of $4 \le D_e/d \le 20$. The radial void fraction correlation is represented by the following principal equation

$$\epsilon_r = \epsilon_o + (1-\epsilon_o)\left\{J_o(\alpha r^*)e^{\beta r^*} + J_o(\alpha[R^* - r^*])e^{\beta(R^*-r^*)}\right\} \quad (1)$$

where

$$\alpha = 6.64 - 6.1e^{-R^*} \quad (2)$$

$$\beta = -0.69 - 0.015R^* \quad (3)$$

$$\epsilon_o = 0.456 - 0.3e^{-R^*} \quad (4)$$

$$r^* = \frac{r}{d} \quad (5)$$

$$R^* = \frac{D_e}{2d} \quad (6)$$

and

$$0 \le r^* \le R^*.$$

Results and Discussion

37 APB radial fraction distributions with diameter aspect ratios ranging from $4 \le D_e/d \le 20$ were generated from the analytical/numerical methods presented by Mueller (1992, 1997). The only error involved in applying these analytical/numerical equations is due to the numerical integration technique needed to completely solve the equations. Although the Gauss-Kronrod method (Davis and Rabinowitz, 1984) is used in this study for the numerical integration, any high-order numerical integration algorithm will provide results with errors that are so small as to be practically zero.

It is well known that the radial void fraction reaches a constant value at a distance of approximately 4–5 particle diameters from the wall of packed beds with diameter aspect ratios greater than 10 (Benenati and Brosilow, 1962; Ridgway and Tarbuck, 1968; Goodling et al., 1983; Govindarao and Froment, 1986). As a result of this property, APBs with a thickness R^* greater than 10 particle diameters ($D_e/d > 20$) are not included in this study since the two walls are far enough apart to act independently and will not simultaneously affect the radial void fraction distribution. In addition, radial void fraction distributions were generated for APBs with a thickness R^* less than two particle diameters ($D_e/d < 4$). However, these void fraction profiles were significantly different from those with R^* greater than two particle diameters and were not included in this investigation.

Mueller (1992) gives a correlation that accurately predicts the radial void fraction in packed beds from a wall of a cylindrical container. Since APBs have two walls, the general form of the radial void fraction correlation given by Eq. 1 was obtained from a modified linear combination of the function given by Mueller (1992). In addition to Eq. 1 other regression equations were evaluated in the analysis. In the evaluation process three criteria were used to assess the predictive value of an equation. First, in order to make the equation useful for modeling purposes the equation had to include as many variables as feasible so that R^2 was as close as possible to the value of one. Secondly, to make the equation useful for predictive purposes the number of variables were held to a minimum. Thirdly, the one principal equation would have to closely predict the void fraction in the entire region between the two walls of the APB. There were several regression equations that met two of the three criteria. However, Eq. 1

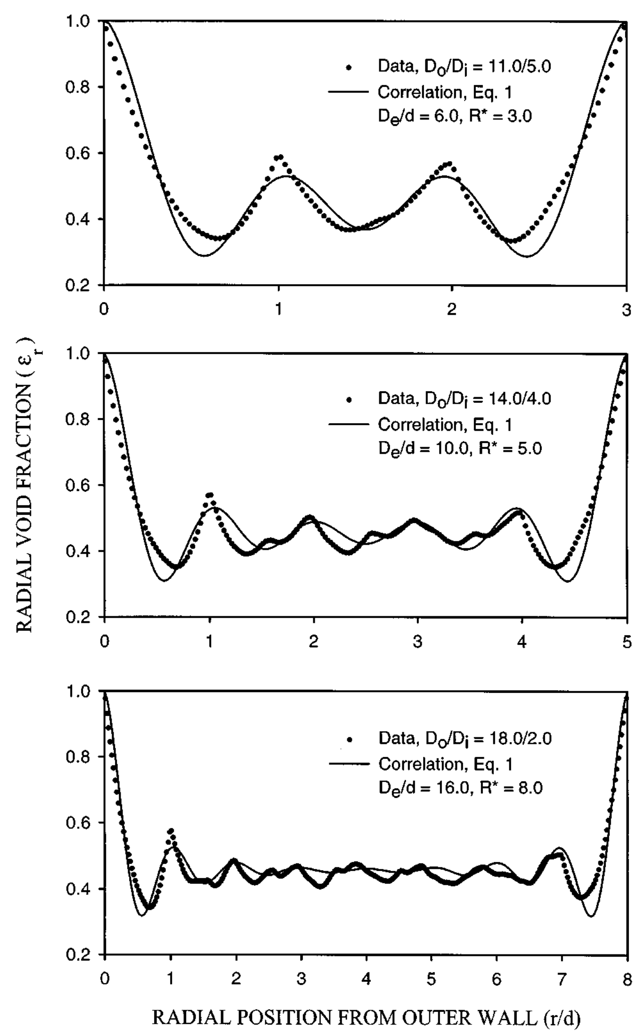

Figure 1. Annular radial void fraction distributions.

was the one that met all three criteria and is capable of modeling all 37 different radial void fraction distributions with R^2 values greater than 90%. The R^2 coefficient for the correlation equation is a measure of the goodness of fit to that of the analytical/numerical distributions. The closer the value of the R^2 coefficient is to 1.0, the better the statistical measure of the fit for the correlation.

In evaluating the coefficients, Eqs. 2–4, the above first two criteria were again used. In selecting the final equations for the coefficients a compromise was used between the opposing requirements of the two criteria. The compromise was to select an equation for a variable that had a minimum number of terms, but still had a reasonably high R^2 value.

Figure 1 shows the results of the analysis for three of the 37 APB radial void fraction distributions. As shown, the two walls of the bed simultaneously affect the radial void fraction distributions for APBs. The solid circular dots represent the distributions obtained from the analytical/numerical method and the solid lines from the correlation given by Eq. 1.

Because of the different curvatures of the two walls and the particular sphere packing that occurs in an APB, the radial void fraction distributions are not symmetrical between the two walls. This non-symmetry becomes less pronounced for small thickness APBs with large bed diameters as compared to small bed diameters. However, the difference is relatively small and the correlation in general closely predicts the radial void fraction profiles for all cases.

In conclusion, this investigation presents radial void fraction distributions for annular packed beds and a correlation equation that can closely predict the radial void fraction as a function of the radial position for diameter aspect ratios of $4 \leq D_e/d \leq 20$.

Notation

J_o = Bessel function of the first kind of order zero
r = radial position from outside wall, m
r^* = dimensionless radial position from outside wall, Eq. 5
α = coefficient, Eq. 2
β = coefficient, Eq. 3
ϵ_o = coefficient, Eq. 4
ϵ_r = radial void fraction, Eq. 1

Literature Cited

Bey, O., and G. Eigenberger, "Fluid Flow Through Catalyst Filled Tubes," *Chem. Eng. Sci.*, **52**, 1365 (1997).

Benenati, R. F., and C. B. Brosilow, "Void Fraction Distribution in Beds of Spheres," *AIChE J.*, **8**, 359 (1962).

Borkink, J. G. H., and K. R. Westerterp, "Significance of the Radial Porosity Profile for the Description of Heat Transport in Wall-Cooled Packed Beds," *Chem. Eng. Sci.*, **49**, 863 (1994).

Cheng, P., and C. T. Hsu, "Fully-Developed, Forced Convective Flow Through an Annular Packed-Sphere Bed with Wall Effects," *Int. J. Heat Mass Transfer*, **29**, 1843 (1986).

Choi, C. Y., and F. A. Kulacki, "Non-Darcian Effects on Mixed Convection in a Vertical Packed-Sphere Annulus," *J. Heat Transfer*, **115**, 506 (1993).

Cohen, Y., and A. B. Metzner, "Wall Effects in Laminar Flow of Fluids Through Packed Beds," *AIChE J.*, **27**, 705 (1981).

Davis, P. J., and R. Rabinowitz, *Methods of Numerical Integration*, 2nd ed., Academic Press, Orlando, FL (1984).

Dean, D. W., "Heat Transfer in Annular Packed Particle Beds," MS Thesis, University of Wisconsin-Madison (1983).

Goodling, J. S., R. I. Vachon, W. S. Stelpfug, S. J. Ying, and M. S. Khader, "Radial Porosity Distribution in Cylindrical Beds Packed with Spheres," *Power Technol.*, **35**, 23 (1983).

Govindarao, V. M. H., and G. F. Froment, "Voidage Profiles in Packed Beds of Spheres," *Chem. Eng. Sci.*, **41**, 533 (1986).

Govindarao, V. M. H., M. Subbanna, A. V. S. Rao, and K. V. S. Ramrao, "Voidage Profile in Packed Beds by Multi-Channel Model: Effects of Curvature of the Channels," *Chem. Eng. Sci.*, **45**, 362 (1990).

Heggs, P. J., D. I. Ellis, and M. S. Ismail, "The Modelling of Fluid-Flow Distributions in Annular Packed Beds," *Gas Sep. Purif.*, **8**, 257 (1994).

Heggs, P. J., D. I. Ellis, and M. S. Ismail, "Prediction of Flow Distributions and Pressure Changes in Multi-Layered Annular Packed Beds," *Gas Sep. Purif.*, **9**, 243 (1995).

Horn, F. L., J. R. Powell, and J. M. Savino, "Particulate Fuel Bed Tests," *Space Nuclear Power Systems 1985*, M. S. El-Genk and M. D. Hoover, eds., Orbit Book Co., Malabar, FL (1986).

Horn, F. L., J. R. Powell, and O. W. Lazareth, "Particle Bed Reactor Propulsion Vehicle Performance and Characteristics as an Orbital Transfer Rocket," *Space Nuclear Power Systems 1986*, M. S. El-Genk and M. D. Hoover, eds., Orbit Book Co., Malabar, FL (1987).

Kubie, J., "Influence of Containing Walls on the Distribution of Voidage Packed Beds of Uniform Sphere," *Chem. Eng. Sci.*, **43**, 1403 (1988).

Kamiuto, K., S. Saito, and K. Ito, "Numerical Model for Combined Conductive and Radiative Heat Transfer in Annular Packed Beds," *Num. Heat Transfer*, **23**, 433 (1993).

Ludewig, H., J. R. Powell, M. Todosow, G. Maise, R. Barletta, and D. G. Schweitzer, "Design of Particle Bed Reactors for the Space Nuclear Thermal Propulsion Program," *Prog. in Nuclear Energy*, **30**, 1 (1996).

Martin, H., "Low Peclet Number Particle-To-Fluid Heat and Mass Transfer in Packed Beds," *Chem. Eng. Sci.*, **33**, 913 (1978).

McWhirter, J. D., M. E. Crawford, D. E. Klein, and T. L. Sanders, "Model for Inertialess Magnetohydrodynamic Flow in Packed Beds," *Fusion Tech.*, **33**, 22 (1998).

Mueller, G. E., "Prediction of Radial Porosity Distributions in Randomly Packed Fixed Beds of Uniformly Sized Spheres in Cylindrical Containers," *Chem. Eng. Sci.*, **46**, 706 (1991).

Mueller, G. E., "Radial Void Fraction Distributions in Randomly Packed Fixed Beds of Uniformly Sized Spheres in Cylindrical Containers," *Powder Tech.*, **72**, 269 (1992).

Mueller, G. E., "Numerical Simulation of Packed Beds with Monosized Spheres in Cylindrical Containers," *Powder Tech.*, **92**, 179 (1997).

Papageorgiou, J. N., and G. F. Froment, "Simulation Models Accounting for Radial Voidage Profiles in Fixed-Bed Reactors," *Chem. Eng. Sci.*, **50**, 3043 (1995).

Powell, J. R., and F. L. Horn, "High Power Density Reactors Based on Direct Cooled Particle Beds," *Space Nuclear Power Systems 1985*, M. S. El-Genk and M. D. Hoover, eds., Orbit Book Co., Malabar, FL (1986).

Ridgway, K., and K. J. Tarbuck, "Voidage Fluctuations in Randomly-Packed Beds of Spheres Adjacent to a Containing Wall," *Chem. Eng. Sci.*, **23**, 1147 (1968).

Rimkevicius, S., V. Survila, and R. Jankauskas, "Heat Transfer and Drag in Axial Flow of Air Through Beds of Spheres Packed into Narrow Annuli," *Heat Transfer-Soviet Res.*, **23**, 814 (1991).

Sodre, J. R., and J. A. R. Parise, "Fluid Flow Pressure Drop Through an Annular Bed of Spheres with Wall Effects," *Exp. Thermal Fluid Sci.*, **17**, 265 (1998).

Song, X., Y. Jin, and Z. Yu, "Influence of Outward Radial Gas Flow on Particle Movement in an Annular Moving Bed," *Powder Tech.*, **79**, 247 (1994).

Subagyo, N. S., and G. A. Brooks, "A New Model of Velocity Distribution of a Single-Phase Fluid Flowing in Packed Beds," *Chem. Eng. Sci.*, **53**, 1375 (1998).

Vortmeyer, D., and R. P. Schuster, "Evaluation of Steady Flow Profiles in Rectangular Circular Packed Beds by a Variational Method," *Chem. Eng. Sci.*, **38**, 1691 (1983).

Vortmeyer, D., and E. Haidegger, "Discrimination of Three Approaches to Evaluate Heat Fluxes for Wall-Cooled Fixed Bed Chemical Reactors," *Chem. Eng. Sci.*, **46**, 2651 (1991).

Yagi, S., and D. Kunii, "Studies on Heat Transfer Near Wall Surface in Packed Beds," *AIChE J.*, **6**, 97 (1960).

Manuscript received Mar. 12, 1999, and revision received Aug. 2, 1999.